Attention and Perception

Lecture 1: Subjective Perception

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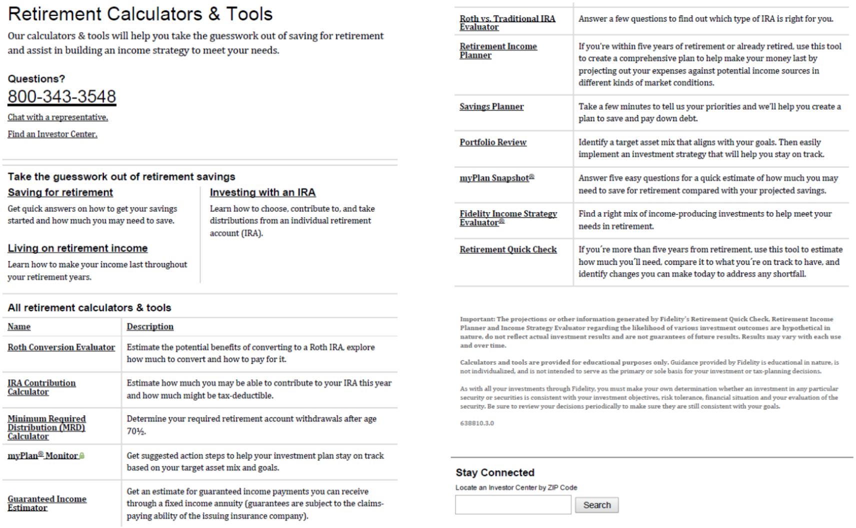
Information and Economic Choices



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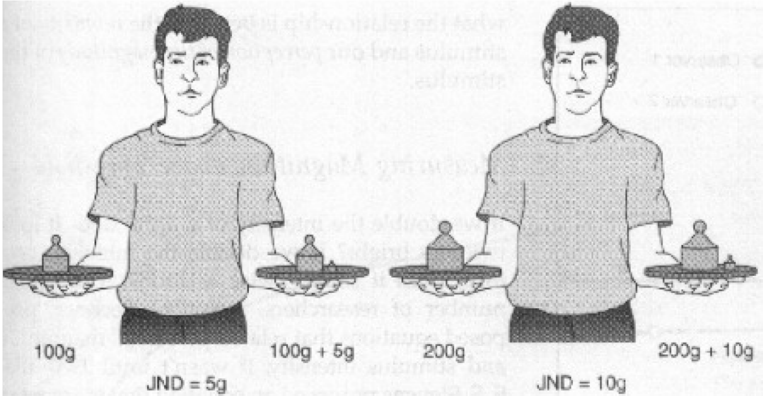
Modeling Challenge

- The simplest model of information search in decision-making is item-by-item search (Caplin, Dean & Martin 2011)
- What to do when item-by-item search is not the appropriate model?
- Could try other forms of information search
    - Such as characteristic-by-characteristic search
- But probably need a fuzzier model of subjective perception

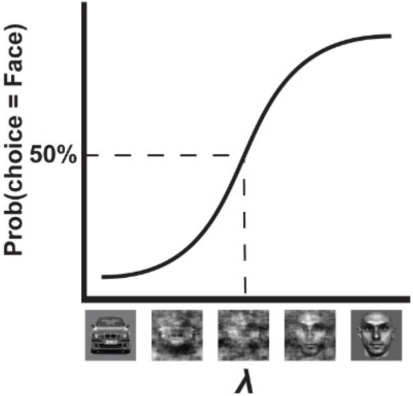
- Weber (1834) reports one of the first modern perception experiments



Weber and Perception

- Weber found the point of just noticeable difference (JND)
- Believed that it was approximately constant
- Theoretical grounding provided by Gustav Fechner (Weber-Fechner law) (Fechner 1860)
- Good approximation:
  - Brightness of lights
  - Intensity and the pitch of sounds
- Bad approximation:
  - Wavelength of light
- Testable implications of JND in consumption provided by Dziewulski (2020)

- Gradual nature of JND illustrated by "psychometric" curves



From Antonio Rangel

State-Dependent Stochastic Choice Data

- Key insight from psychometric analysis: choices are inherently stochastic and state dependent
- Examples:
  - Probability chose left hand when heavier weight is in the left hand
  - Probability detect blue when the circle is X% blue
  - Probability say dots moving left when the dots are moving left

Perception Today

- In psychology and neuroscience, subjective perception continues to be central
  - See McFadden (1999) review
- Canonical perception task is moving dots or coherent motion task
  - For example Roitman & Shadlen (2002)
- Guess which direction dots are moving on a screen (*left* or *right*)
  - Ask monkeys (juice rewards) or humans
- Only some proportion of the dots move in a coherent direction

Click here for example

Perception Today

- How do psychologists and neuroscientists analyze subjective perception of dots?
- Assume model (functional form) of how decision makers perceive the dots
    - Leading models: drift diffusion model, race model, signal detection theory
- Estimate model parameters from observables
    - Choices, brain activity, decision times, eye-tracking, etc.

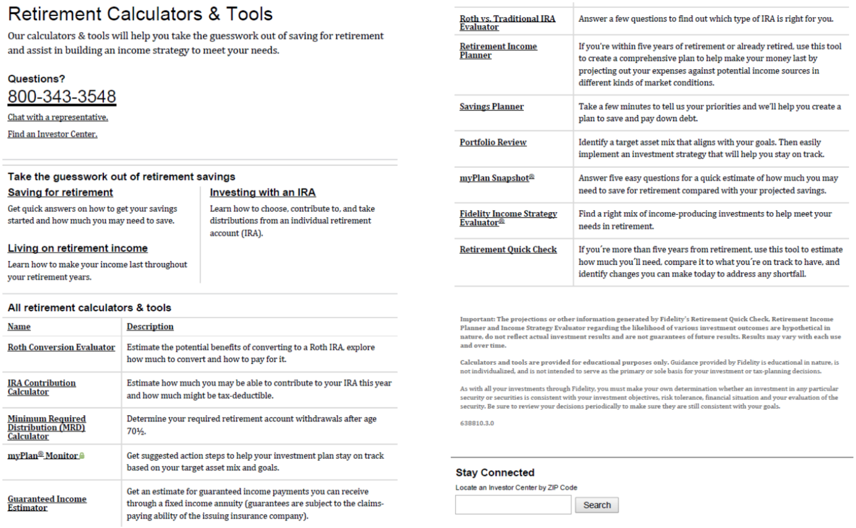
Perception Today

- Standard assumptions for models of subjective perception:
  - Decision makers get evidence/signals about the world
  - Update prior beliefs according to Bayes' Rule
  - Use posterior beliefs to maximize expected value/utility
- Also standard in economics, computer science, information theory, etc.
- Will consider deviations later in class
- But for now, we stay on the main road for models of subjective perception
  - We will consider it as an "as if" model of how decision-makers process readily-available information

Subjective Perception and Economic Decision Making

- There are many facts about the world that can be payoff-relevant
  - Fundamentals of a stock, the effectiveness of a vaccine, characteristics of a health plan, etc.
- Increasing evidence that individuals can be poorly informed about these facts when making consequential decisions
- Many reasons why poorly informed:
  - Only receive partial information from firms or policymakers
  - Only internalize some of the available information
  - Only recall some of this information at moment of choice

Recall



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Perspective, Approach, Questions

- My co-authors and I take the perspective of an econometrician who only observes a decision maker's (DM's) choices, not how well informed they are
- Why? Because the DM's perception is subjective (the information that is internalized is private)
- We employ an "information-theoretic revealed preference" approach
  - Assume standard information theory framework: correct prior μ^* → signal realization → update beliefs correctly → maximize expected utility
  - But treat utility, signals, and beliefs as unobservable!
- Key questions:
  - Consistency: *As if* the DM chooses optimally given some private information?
  - Recovery: What does the DM value and how valuable is their private information?

Caplin & Martin (2015), "A Testable Theory of Imperfect Perception."

- Question: Can we assume that a decision maker is **updating beliefs correctly** (using Bayes' Rule) and **maximizing utility** for unobservable private information?
    - Key assumptions behind many, if not most, models of perception
    - Includes: drift diffusion model, race model, signal detection theory
    - Also captures unobserved persuasion (not know what advisor says to client)

- Use choice data (observable) to understand private information (unobservable)

- In other words: We consider an econometrician who wants to determine if a decision maker chose optimally given some private information, but only knows the actions taken, and nothing about information obtained
- For example, all the econometrician might know about the decision-maker's information is that it produces the following joint distribution over actions (a_1, a_2) and states (ω_1, ω_2)

$$P = \begin{array}{cc} \begin{array}{cc} \omega_1 & \omega_2 \end{array} & \\ \begin{pmatrix} .4 & .1 \\ .1 & .4 \end{pmatrix} & \begin{array}{c} a_1 \\ a_2 \end{array} \end{array}$$

Examples of P

- Probability buy a stock when it has certain fundamentals
- Probability PPO health plan is chosen when it has certain benefits
- Probability buy fresh bread when packaged bread is at a certain price
- Probability hire an employee who will be rated highly
- Probability make a loan to someone who will default
- Probability of ordering a test for someone who has a disease
- Common elements: State impacts prizes, decision maker may not be fully informed about state, analyst knows state given data or time

Knowns

- Example:

$$P = \begin{array}{cc} \omega_1 & \omega_2 & \\ \left(\begin{array}{cc} .4 & .1 \\ .1 & .4 \end{array}\right) & & \begin{array}{c} a_1 \\ a_2 \end{array} \end{array}$$

- Actions A (finite)
- States Ω (finite)
- P: Joint distribution of actions and states
  - $P(a, \omega)$ is the probability of taking action a and being in state ω
  - Objective prior over states given by $\mu^*(\omega) = \sum_{a \in A} P(a, \omega)$
  - Assume all actions chosen to avoid minutiae
  - Note: Caplin & Martin (2015) (CM15 hereafter) uses q instead of P

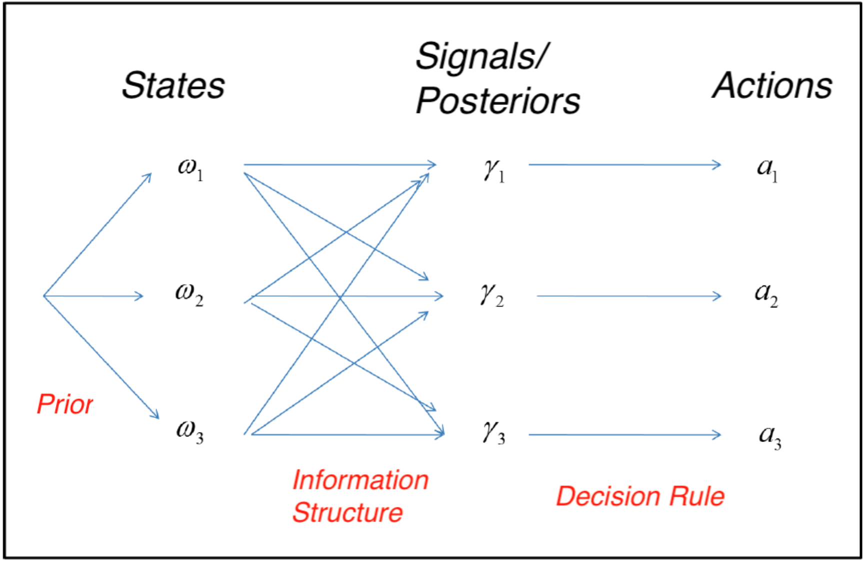
Possibly known

- Prizes X
  - $x(a, \omega) \in X$ is the prize from taking action a in state ω
  - Utility function defined over this set: $u : X \to \mathbb{R}$
  - Note: Caplin & Dean (2015) (CD15 hereafter) and Caplin, Dean & Leahy (2022) (CDL22 hereafter) use compact $a(\omega)$
- Example: Good prize for matching state to action (tracking problem)

$$x(a, \omega) = \begin{array}{cc} \omega_1 & \omega_2 \\ \end{array} \begin{pmatrix} x_G & x_B \\ x_B & x_G \end{pmatrix} \begin{array}{c} a_1 \\ a_2 \end{array} \quad \text{so can normalize to} \quad u = \begin{array}{cc} \omega_1 & \omega_2 \\ \end{array} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{array}{c} a_1 \\ a_2 \end{array}$$

- If prizes are unknown, WLOG to assume that a distinct prize is obtained from taking each action in each state

- Unknown (not observed):
  1. **Utility function** $u : X \to \mathbb{R}$
     - X is the set of prizes, and $x(a, \omega)$ is the prize from taking action a in state ω
  2. **Subjective prior** $\mu : \Omega \to (0, 1)$
     - CM15 and CD15 treat this as observable
  3. **Information structure** $\pi : \Omega \to \Delta(\Gamma)$
     - "Posterior-based" approach of Bayesian persuasion (Kamenica & Gentzkow 2011): π maps into distributions of posteriors $\gamma \in \Gamma$
     - Can think of these posteriors as being generated by an underlying signal structure S
     - $\pi(\gamma|\omega)$ is the probability of posterior γ in state ω
     - $\pi(\gamma)$ is the unconditional probability of posterior γ
     - Note: CDL22 use Q and call "attention strategy"
  4. **Decision rule** $\sigma : \Gamma \to \Delta(A)$
     - $\sigma(a|\gamma)$ probability of action a given posterior γ
     - Note: CM15 and CD15 use C and call "choice function", CDL22 use q and call "action strategy"


States
Signals/ Posteriors
Actions
ω_1
ω_2
ω_3
γ_1
γ_2
γ_3
a_1
a_2
a_3
Prior
Information Structure
Decision Rule

Consistency

- Question: Is a given *P* "consistent" with Bayes' Rule and expected utility maximization (BEU)?
- In other words, do there exist unobservables (u, μ, π, σ) that could explain what we see in *P* and are in line with Bayes' Rule and expected utility maximization?
- Exercise: Find (u, μ, π, σ) that could explain what we see in this *P* and are in line with Bayes' Rule and expected utility maximization in this tracking problem

$$u = \begin{array}{cc} \omega_1 & \omega_2 \\ \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right) & \begin{array}{c} a_1 \\ a_2 \end{array} \end{array} \quad \& \quad P = \begin{array}{cc} \omega_1 & \omega_2 \\ \left(\begin{array}{cc} .4 & .1 \\ .1 & .4 \end{array}\right) & \begin{array}{c} a_1 \\ a_2 \end{array} \end{array}$$

Consistency

- For a utility function u, it's easy to assess if data P is not consistent with the model
- Not consistent if can improve utility by making a *wholesale* switch from any chosen action to another action
  - In other words, if can improve utility by switching to action a_2 at all posteriors where chose action a_1
  - This means that whatever the decision rule is, it doesn't maximize expected utility
- The following simple example illustrates

- Example:

$$u = \begin{array}{cc} \omega_1 & \omega_2 & \\ \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right. & \left. \right) & \begin{array}{c} a_1 \\ a_2 \end{array} \end{array}$$

$$u = \begin{array}{c|c} & \begin{array}{cc} \omega_1 & \omega_2 \end{array} \\ \end{array}$$

- For this utility function, P is not consistent with any information structure
- Why? Can improve utility by making a *wholesale* switch to choosing a_2 when chose a_1
  - When chose a_1, got 1 with unconditional probability .1, but if had chosen a_2 instead, could have gotten 1 with unconditional probability .4
- Now introduce general version of above logic: restrictions ensuring no utility increase from making wholesale switch from any chosen action

Definition

(u, μ, π, σ) is a **BEU representation** of P if it satisfies:

1. Data Matching[1]:

$$P(a, \omega) = \mu(\omega) \sum_{\gamma \in \Gamma} \pi(\gamma|\omega)\sigma(a|\gamma)$$

2. Bayesian Updating:

$$\gamma(\omega) = \frac{\mu(\omega)\pi(\gamma|\omega)}{\sum_{v \in \Omega} \mu(v)\pi(\gamma|v)}$$

3. Maximization: If $\pi(\gamma) > 0$ and $\sigma(a|\gamma) > 0$,

$$\sum_{\omega \in \Omega} \gamma(\omega) u(x(a, \omega)) \geq \sum_{\omega \in \Omega} \gamma(\omega) u(x(b, \omega))$$

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[1] Note that Data Matching implies $\mu = \mu^*$.

Definition

Utility function u satisfies the No Improving Action Switches (**NIAS**) condition with respect to P if,

$$\sum_{\omega\in\Omega} P(a,\omega)u(x(a,\omega)) \geq \sum_{\omega\in\Omega} P(a,\omega)u(x(b,\omega)),$$

for all $a, b \in A$.

Intuition: Cannot improve utility by making a *wholesale* switch from any chosen action

Theorem

P has a BEU representation if and only if there exists $u : X \to \mathbb{R}$ satisfying the NIAS inequalities.

Note: To rule out global indifference, just require the inequality to be strict for some $a, b \in A$ in Maximization and NIAS

- Illustration of NIAS inequalities:
    - Tracking problem:

$$u = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \omega_1 & \omega_2 \\ \left(\begin{array}{c} 1 \\ 0 \end{array}\right. & \left.\begin{array}{c} 0 \\ 1 \end{array}\right) \end{array} \begin{array}{c} \\ a_1 \\ a_2 \end{array}$$

    - Objective prior $\mu^*(\omega_1) = 75\%$
    - Choose correctly around 55% of the time in both states (a_1 in ω_1 and a_2 in ω_2)
- Pause: Can we model this decision maker as updating beliefs correctly and maximizing expected utility given their imperfect perception?
- **NIAS shows cannot be updating beliefs correctly and maximizing expected utility given imperfect perception!**

- Recall no EU improvement from switching from action a to action b:

$$\sum_{\omega \in \Omega} P(a, \omega) u(x(a, \omega)) \geq \sum_{\omega \in \Omega} P(a, \omega) u(x(b, \omega))$$

- NIAS are simply:

$$\begin{aligned} \text{For action } a_1 &: P(a_1, \omega_1) \geq P(a_1, \omega_2) \\ \text{For action } a_2 &: P(a_2, \omega_2) \geq P(a_2, \omega_1) \end{aligned}$$

- Or:

$$\begin{aligned} \text{For action } a_1 &: P(a_1 | \omega_1) \mu^*(\omega_1) \geq P(a_1 | \omega_2) \mu^*(\omega_2) \\ \text{For action } a_2 &: P(a_2 | \omega_2) \mu^*(\omega_2) \geq P(a_2 | \omega_1) \mu^*(\omega_1) \end{aligned}$$

- In this example:

$$\begin{aligned} \text{For action } a_1 &: .55 * .75 \geq .45 * .25 \\ \text{For action } a_2 &: .55 * .25 \not\geq .45 * .75 \end{aligned}$$

- NIAS provides a simple test for optimal choice with private information
  - Look for solution to a set of linear inequalities
  - Easier to implement than test of Lu (2016)
- NIAS has been tested in lab experiments using perception tasks (later)
  - Sums of numbers (Dean & Neligh 2023, Almog & Martin 2024)
  - Proportion of colored balls (Dean & Neligh 2023, de Clippel & Rozen 2021)
  - Number of balls and types of angles (Dewan & Neligh 2020)
  - Counting shapes (Caplin, Csaba, Leahy & Nov 2020)
- Was tested in real-world setting with limited data by Rambachan (2024)
- Was extended to settings without state observability by Rehbeck (2023)

- Theoretical applications in Caplin & Martin (2015):
  1. Utility recovery
     - In subsequent JPE paper, provide the convex geometry to actually do this (Caplin & Martin 2021)
  2. Robust predictions
     - Conceptually and technically related to robustness agenda of Bergemann and Morris (Bergemann & Morris 2013, 2016)
  3. Model classification
     - Example: Can Logit model be explained with imperfect perception?

- Consider a standard tracking problem with actions: {l,r} and states: {L,R}:

$$u = \begin{array}{cc} \omega_1 = L & \omega_2 = R & \\ \left(\begin{array}{c} 1 \\ 0 \end{array} \right. & \left. \begin{array}{c} 0 \\ 1 \end{array} \right) & \begin{array}{l} a_1 = l \\ a_2 = r \end{array} \end{array}$$

- Pause: Can you work out the NIAS conditions for this setting?
- Recall no EU improvement from switching from action a to action b:

$$\sum_{\omega \in \Omega} P(a,\omega) u(x(a,\omega)) \geq \sum_{\omega \in \Omega} P(a,\omega) u(x(b,\omega))$$

- They are simply:

$$\text{For action } l : P(l,L) \geq P(l,R)$$
$$\text{For action } r : P(r,R) \geq P(r,L)$$

- Logit generates the following data:

$$P(I|L) = \frac{e^{\frac{1}{\lambda}}}{1+e^{\frac{1}{\lambda}}} \ \ \& \ \ P(I|R) = \frac{1}{1+e^{\frac{1}{\lambda}}}$$

- Pause: Can you work out the implications given NIAS?
- They are:

$$\frac{e^{\frac{1}{\lambda}}}{1+e^{\frac{1}{\lambda}}} P(L) \geq \frac{1}{1+e^{\frac{1}{\lambda}}} (1-P(L))$$
$$\frac{e^{\frac{1}{\lambda}}}{1+e^{\frac{1}{\lambda}}} (1-P(L)) \geq \frac{1}{1+e^{\frac{1}{\lambda}}} P(L)$$

- For $P(L) \neq 1/2$, NIAS shows Logit model can be explained with imperfect perception if and only if,

$$\lambda \leq \frac{1}{\left|\ln\left(\frac{P(L)}{1-P(L)}\right)\right|}$$

Costly Attention Applications

- NIAS is also a foundational condition for models of costly attention
  - Necessary and sufficient condition for choosing optimally given chosen attention
- NIAS+NIAC ⇔ rational inattention (Caplin & Dean 2015) (later)
- NIAS+GACI ⇔ non-separable costs (Chambers, Liu & Rehbeck 2020)
- NIAS+Axiom 3 ⇔ posterior-separable costs (Denti 2022)

Open Questions

- NIAS w/ unobservable preference heterogeneity
    - Analogous to recent work on unobservable preferences and consideration sets
    - Make falsifiable by putting functional form restrictions on preferences and/or by restricting to a class of problems?
    - Challenge for NIAS: Can characterize restrictions for each posterior, but cannot cleanly aggregate over unobserved posteriors that generate same action
    - Could also put assumptions on prior and/or costs of information (link w/ Brown & Jeon (2024))
- NIAS with belief biases (link w/ de Clippel & Zhang (2022))
    - Need either if NIAS fails or if in setting where belief biases are plausible
    - For example, incorrect priors/stereotyping in discrimination work

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