Attention and Perception

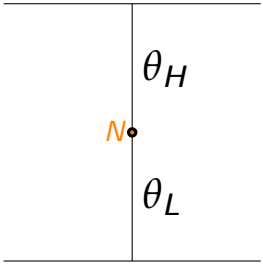
Lecture 8: Rational Inattention in Games (Theory)

Daniel Martin
danielmartin@ucsb.edu

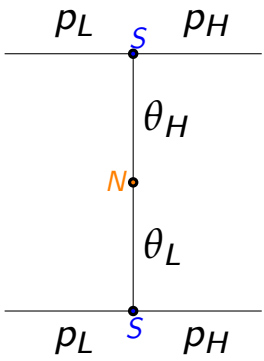
Market Setting

- Figuring out the quality of products requires cognitive effort
- But if consumers are inattentive to quality, firms with low-quality products will be able to trick consumers into paying the same price as high-quality products
- In Martin (2017), I used a very simple setting to capture this tension
  - One buyer, one seller, one product, static environment

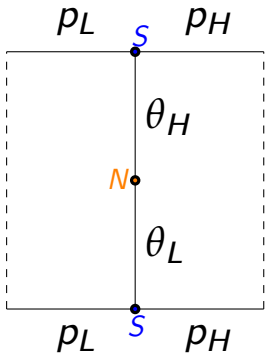
- Nature draws quality $\theta \in \{\theta_L, \theta_H\} \subset \mathbb{R}_+$
- Prior probability of high quality $\mu_0 \equiv \Pr(\theta_H)$



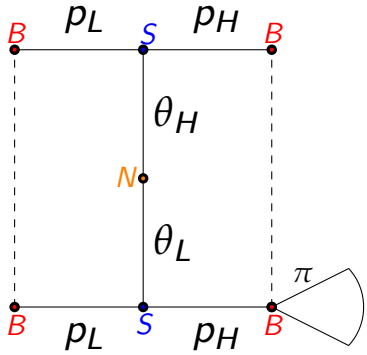
- Seller observes θ and chooses price $p \in \{p_L, p_H\} \subset \mathbb{R}_+$
- Model generalizes to many prices



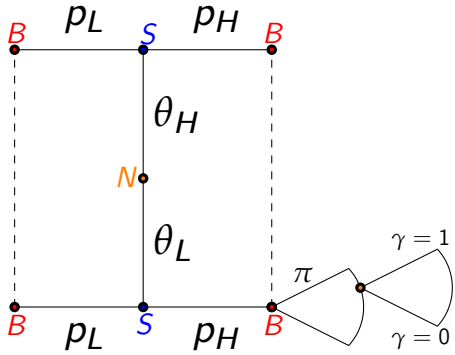
- Buyer observes p and forms interim belief of high quality $\mu_p \equiv \Pr(\theta_H \mid p)$, combining prior μ_0 and seller strategy



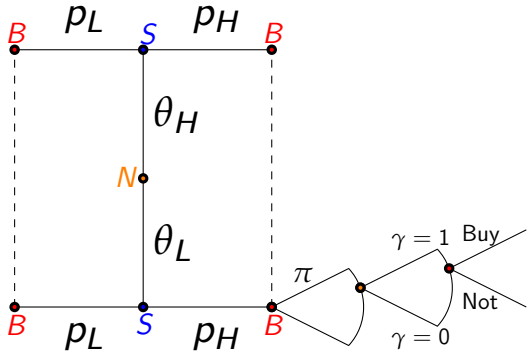
- Buyer selects an *attention technology* $\pi : \{\theta_L, \theta_H\} \to \Delta(\Gamma)$, with $\Gamma = [0, 1]$, that satisfies Bayes' rule and has finite support



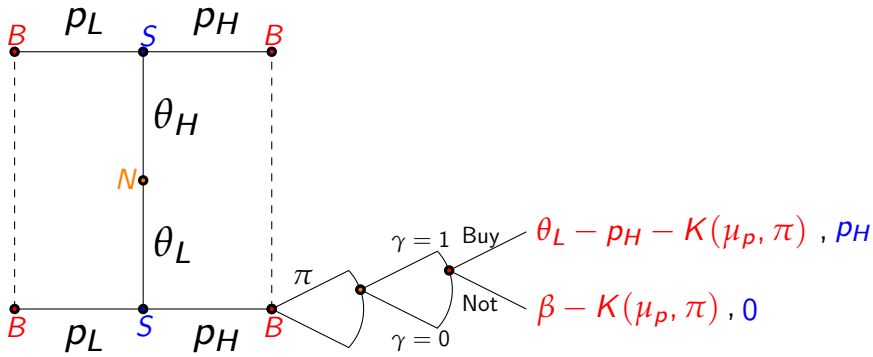
- Nature realizes a posterior belief of high quality $\gamma \in [0, 1]$



- Buyer decides whether to purchase one unit of the good



- Standard utility function for buyer and seller
- Buyer's outside option $\beta > 0$, attention cost function K



Buyer's Attention

- Shannon cost of attention technology π, given interim belief μ_p:

$$K(\mu_p, \pi) = \kappa \sum_{\gamma \in \Gamma} \pi(\gamma) \Big(\gamma \ln \gamma + (1-\gamma) \ln(1-\gamma) \Big) - \kappa \Big(\mu_p \ln \mu_p + (1-\mu_p) \ln(1-\mu_p) \Big)$$

- Two useful properties greatly simplify analysis: separability of the prior term and the log-linear form
- Note: seller actions influence the interim belief μ_p

Equilibrium

There are only two mixed-strategy PBE under rational inattention:

Pooling-Low

High- and low-quality sellers both charge p_L with probability 1. Buyer never acquires information and holds pessimistic beliefs about high prices ("strategic ignorance").

Mimic-High

High-quality sellers charge p_H with probability 1. Low-quality sellers mimic and charge p_H with probability η. Buyer typically attends when $p = p_H$.

Captures strategic tension of interest

This is the equilibrium observed in the lab

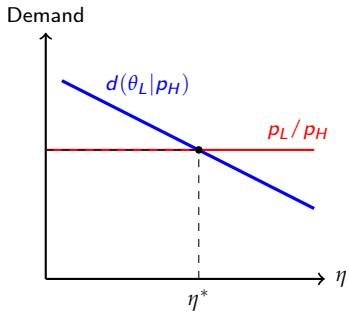
Theorem (Existence & Uniqueness of η)

For any game G there exists an equilibrium of the *mimic-high* type in which high-quality sellers set p_H with prob. 1 and low-quality sellers set p_H with prob. $\eta \in [0, 1]$, where η is uniquely pinned down.

Key: a seller of type θ faces buyer demand $d(\theta|p)$ when setting price p, so the low-quality seller indifference condition is

$$d(\theta_L|p_H)\, p_H \;=\; p_L \quad\Longrightarrow\quad d(\theta_L|p_H) \;=\; \frac{p_L}{p_H}$$

Because $d(\theta_L|p_H)$ is strictly decreasing in η (single-crossing), the solution is unique



Let γ^0 (don't buy) and γ^1 (buy) denote the posterior cut-offs at p_H

Rational inattention implies

$$\ln\frac{\gamma^1}{\gamma^0} = \frac{(\theta_H - p_H) - \beta}{\kappa} \qquad \& \qquad \ln\frac{1-\gamma^1}{1-\gamma^0} = \frac{(\theta_L - p_H) - \beta}{\kappa}$$

Property: thresholds do *not* depend on the prior μ_p (Caplin, Dean & Leahy 2022)

So conditional demand

$$d(\theta_L|p_H) = \frac{\left(\frac{p_H-\beta}{1-\gamma^1}\right)(\mu_{p_H} - \gamma^0)}{(1-\mu_{p_H})}$$

is strictly increasing in μ_{p_H} and hence strictly *decreasing* in η

Experimental Test

- My JMP paper included a lab experiment run at NYU
- The experiment confirmed that subjects play the mimic-high equilibrium ("equilibrium selection"), but it had low power and no treatment variation
- I removed the experiment and published the theory paper at GEB
- A decade later my student David Almog and I redesigned it as a stronger/better test of RI in games...

References I

Caplin, A., Dean, M. & Leahy, J. (2022), 'Rationally inattentive behavior: Characterizing and generalizing Shannon entropy', *Journal of Political Economy* **130**(6), 1676–1715.

Martin, D. (2017), 'Strategic pricing with rational inattention to quality', *Games and Economic Behavior* **104**, 131–145.