Attention and Perception

Lecture 9: Costly Attention or Experiments: NIAC vs. NIAC$^+$

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Example 1

Decision Problem 1

Action	**Payoff 49 red balls**	**Payoff 51 red balls**
a_1	10	0
b_1	0	10

Prior: $\{0.5, 0.5\}$

Action	**State = 49 red balls**	**State = 51 red balls**
Prob choose a_1	$\frac{3}{4}$	$\frac{1}{4}$
Prob choose b_1	$\frac{1}{4}$	$\frac{3}{4}$

Example 1

Decision Problem 2

Action	**Payoff 49 red balls**	**Payoff 51 red balls**
a_2	20	0
b_2	0	20

Prior: $\{0.5, 0.5\}$

Action	**State = 49 red balls**	**State = 51 red balls**
Prob choose a_2	$\frac{2}{3}$	$\frac{1}{3}$
Prob choose b_2	$\frac{1}{3}$	$\frac{2}{3}$

Testing NIAC

- No Improving Attention Cycles (NIAC) is evaluated prior-by-prior
  - Why? Because attention costs depend on the prior: $K(\mu, \pi)$
  - Why? Because the prior determines the posterior on which costs are based
- NIAC can be tested with Decision Problems 1 and 2 because the prior is the same
- We test NIAC by looking at the impact of cycling revealed information structures (which are given by P_1 and P_2)

$$P_1 = \begin{array}{cc} 49 & 51 \\ \left(\begin{array}{cc} \frac{3}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{3}{8} \end{array}\right) & \end{array} \begin{array}{c} a_1 \\ b_1 \end{array} \quad \& \quad P_2 = \begin{array}{cc} 49 & 51 \\ \left(\begin{array}{cc} \frac{2}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{2}{6} \end{array}\right) & \end{array} \begin{array}{c} a_2 \\ b_2 \end{array}$$

The Value of Information

- The value of the information structure used in decision problem 1, evaluated in decision problem 1, is

$$\max_{a\in\{a_1,b_1\}}[P_1(a_1,49)u(a,49)+P_1(a_1,51)u(a,51)]+\max_{a\in\{a_1,b_1\}}[P_1(b_1,49)u(a,49)+P_1(b_1,51)u(a,51)]$$

$$\frac{3}{8}10+\frac{1}{8}0+\frac{1}{8}0+\frac{3}{8}10=\frac{60}{8}=7\frac{1}{2}$$

- Switching to the information structure used in decision problem 2, evaluated in decision problem 1, gives

$$\max_{a\in\{a_1,b_1\}}[P_2(a_2,49)u(a,49)+P_2(a_2,51)u(a,51)]+\max_{a\in\{a_1,b_1\}}[P_2(b_2,49)u(a,49)+P_2(b_2,51)u(a,51)]$$

$$\frac{2}{6}10+\frac{1}{6}0+\frac{1}{6}0+\frac{2}{6}10=\frac{40}{6}=6\frac{2}{3}$$

The Value of Information

- The value of the information structure used in decision problem 2, evaluated in decision problem 2, is

$$\max_{a\in\{a_2,b_2\}}[P_2(a_2,49)u(a,49)+P_2(a_2,51)u(a,51)]+\max_{a\in\{a_2,b_2\}}[P_2(b_2,49)u(a,49)+P_2(b_2,51)u(a,51)]$$

$$\frac{2}{6}20+\frac{1}{6}0+\frac{1}{6}0+\frac{2}{6}20=\frac{80}{6}=13\frac{1}{3}$$

- Switching to the information structure used in decision problem 1, evaluated in decision problem 2, gives

$$\max_{a\in\{a_2,b_2\}}[P_1(a_1,49)u(a,49)+P_1(a_1,51)u(a,51)]+\max_{a\in\{a_2,b_2\}}[P_1(b_1,49)u(a,49)+P_1(b_1,51)u(a,51)]$$

$$\frac{3}{8}20+\frac{1}{8}0+\frac{1}{8}0+\frac{3}{8}20=\frac{120}{8}=15$$

NIAS and NIAC

- P_1 and P_2 satisfy NIAS because no wholesale action switch improves payoffs
- To check NIAC, note that the change from switching in the first problem is $6\frac{2}{3} - 7\frac{1}{2}$ and the change from switching in the second problem is $15 - 13\frac{1}{3}$, so there is a net improvement
- This means that this dataset does not satisfy NIAC, so there is no K that rationalizes it

Example 2

Decision Problem 1

Action	**Payoff 49 red balls**	**Payoff 51 red balls**
a_1	10	0
b_1	0	10

Prior: $\{0.45, 0.55\}$

Action	**State $=$ 49 red balls**	**State $=$ 51 red balls**
Prob choose a_1	$\frac{3}{4}$	$\frac{1}{4}$
Prob choose b_1	$\frac{1}{4}$	$\frac{3}{4}$

Example 2

Decision Problem 2

Action	**Payoff 49 red balls**	**Payoff 51 red balls**
a_2	20	0
b_2	0	20

Prior: {0.55, 0.45}

Action	**State = 49 red balls**	**State = 51 red balls**
Prob choose a_2	$\frac{2}{3}$	$\frac{1}{3}$
Prob choose b_2	$\frac{1}{3}$	$\frac{2}{3}$

NIAC and NIAC$^+$

- This dataset trivially passes NIAC because it passes within each prior!
- But it will fail NIAC$^+$, which implements a similar test to NIAC but across decision problems with different priors
- NIAC$^+$ in words: rotate the conditional $P_i(a \mid \omega)$ instead of the whole joint distribution $P_i(a, \omega)$
  - NIAC$^+$ is a more demanding test when priors vary and is equivalent to NIAC when they do not
- Intuition: just rotate the signals across decision problems

NIAC$^+$ and the Value of Information

- First, value the *signal* structure used in decision problem 2 at the prior and payoffs from decision problem 2:

$$P_2(a_2 \mid 49)\mu_2^*(49)u(a_2,49) + P_2(a_2 \mid 51)\mu_2^*(51)u(a_2,51)$$
$$+P_2(b_2 \mid 49)\mu_2^*(49)u(b_2,49) + P_2(b_2 \mid 51)\mu_2^*(51)u(b_2,51)$$

- Then switch to the *signal* structure used in decision problem 1, still evaluated with decision problem 2's prior and payoffs:

$$P_1(a_1 \mid 49)\mu_2^*(49)u(a_2,49) + P_1(a_1 \mid 51)\mu_2^*(51)u(a_2,51)$$
$$+P_1(b_1 \mid 49)\mu_2^*(49)u(b_2,49) + P_1(b_1 \mid 51)\mu_2^*(51)u(b_2,51)$$

NIAS, NIAC, NIAC$^+$

- If the data satisfy NIAS and NIAC, then they are consistent with maximizing the general rational inattention model $G_i(\pi) - K(\mu_i, \pi)$
- If the data satisfy NIAS and NIAC$^+$, then they are consistent with maximizing a general costly signal structure model: $G_i(\pi) - K(S)$
  - Recall that S is the signal structure that generated π
  - Why $K(S)$ and not $K(\mu_i, \pi)$? Cost is on the signals generated, not on the posteriors realized (the same signal can generate different posteriors, and different signals can generate the same posterior)
  - Denti, Marinacci & Rustichini (2022) call this the costly "experiments" model
- In de Clippel & Rozen (2021) and Almog & Martin (2024), NIAC$^+$ is satisfied
  - Makes sense because strategic changes in prior do not impact the signal cost (the cost of actually adding up the balls or numbers)
  - An issue with testing NIAC$^+$ in these settings is that strategic priors may be incorrect

Dean and Neligh Prior Experiment

- Dean & Neligh (2023) offers a decision-theoretic setting with clearly specified priors where NIAC^+ can be directly tested
  - Binary matching payoff: a pays 10 in state 1, b pays 10 in state 2
  - Experiment 3 varies the prior across decision problems 7–10

DP	7	8	9	10
$\mu^*(1)$	0.50	0.60	0.75	0.85
$P(a \mid 1)$	0.7663	0.8754	0.8960	0.9073
$P(a \mid 2)$	0.2887	0.3805	0.4023	0.5126

- NIAC has no bite here because the general RI cost can depend on the prior
- NIAC^+ has bite because it rotates the signal structure across priors

Dean and Neligh: NIAC$^+$ Results

	S_7	S_8	S_9	S_{10}
$DP7$	7.3882	7.4743	7.4683	6.9732
$DP8$	7.4432	7.7303	7.7665	7.3931
$DP9$	7.5258	8.1142	8.2140	8.0230
$DP10$	8.5000	8.5000	8.5123	8.5000

- In the dots/balls task, all NIAC$^+$ cycle inequalities are satisfied in aggregate data
  - Smallest slack is 0.0635, from $DP8 \to DP9 \to DP8$
  - But "close" to failing: bootstrap puts 47.98% of minimum slacks below zero
- The equation task passes with more room in aggregate data
  - Smallest slack is 0.6542
  - Only 0.04% of bootstrap minimum slacks are below zero

Shannon Costs Need Not Imply NIAC$^+$

- A Shannon RI model can satisfy local restrictions and still fail NIAC$^+$
  - Two states L, R, two actions ℓ, r, payoffs R_i to matching, $\kappa = 0.5$
  - $\mu_1^*(L) = 0.05,\ R_1 = 3;\ \mu_2^*(L) = 0.10,\ R_2 = 2$
  - Shannon-optimal signal structures and gross values:

	$S(\ell \mid L)$	$S(\ell \mid R)$
S_1	0.9529	0.000124
S_2	0.8354	0.001700

	S_1	S_2
$G_1(\cdot)$	2.9926	2.9705
$G_2(\cdot)$	1.9904	1.9640

- NIAC$^+$ would require one fixed experiment-cost gap:

$$G_1(S_1) - G_1(S_2) = 0.0221 \quad \text{and} \quad G_2(S_1) - G_2(S_2) = 0.0263$$

- The implied cycle slack is $0.0221 - 0.0263 = -0.0042 < 0$

A Nearby Shannon Example That Passes

- The failure is not automatic for Shannon data
  - Keep $\kappa = 0.5$, $\mu_1^*(L) = 0.05$, $R_1 = 3$, and $\mu_2^*(L) = 0.10$
  - Lower the second payoff scale from $R_2 = 2$ to $R_2 = 1.5$

	$S(\ell \mid L)$	$S(\ell \mid R)$
S_1	0.9529	0.000124
S_2	0.5533	0.003061

	S_1	S_2
$G_1(\cdot)$	2.9926	2.9243
$G_2(\cdot)$	1.4928	1.4289

- The NIAC$^+$ slack is $0.0683 - 0.0639 = 0.0044 > 0$
- A fixed experiment-cost gap can rationalize both choices:

$$0.0639 \leq K(S_1) - K(S_2) \leq 0.0683$$

- Intuition: problem 1 is now willing to pay slightly more for S_1 than problem 2 is

References I

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